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Abstract: The present study intends to demonstrate that there is no logical-formal inconsistency in the Christian Trinity. However, the demonstration requires specific tools, other than those of classical logic. There are many older or newer attempts that try to remove the thesis of the inconsistency of the Christian Trinity. There is often a call for mathematical tools. As far as we are concerned, we will appeal to co-inherence and the nesting relationships specific to the Christian Trinity, as they appear especially in Augustine's work. We advance the hypothesis that Augustine's metaphor "heaven of heavens" has a foundational role in the logical plane of explanation. In this sense, Augustine points out that in the "heaven of heavens", reason does not know "in part", but it knows everything suddenly, entirely, as in a totality. This totality with a founding role functions as a principle, which we can call the principle of free totality (PFT). But the co-belonging of entities in the free and founding totality also expresses co-inherence (Leibniz). Divine persons are an emanation of God. That is why we are talking about co-inherence. The "co" particle points out that the starting point is in the free totality of God, and that this totality logically precedes the rest of the process. Thus, we can consider the term "the Christian Trinity" (in its plural sense) as a nested term, as co-habitation. The totality of God is not a generalization in the spirit of the Aristotelian abstraction, but rather a particularity, a "personalization" that does not cancel the individual, but it highlights in instantiation. In this sense, the Christian God is not an abstraction, but a divine Being in three persons, in whom He instantiates. The logic of this process is the one of vagueness.

Key words: relation of nesting, Christian Trinity, logic of the Trinity, free totality, relation of co-inherence, logic of vague
"Which of us comprehendeth the Almighty Trinity? and yet which speaks not of it, if indeed it be It? Rare is the soul, which while it speaks of It, knows what it speaks of. And they contend and strive, yet, without peace, no man sees that vision."

- Augustine

1. Introduction

Any profound philosophy starts from a paradox or ends in a paradox. So is, for example, Kant's philosophy of the "thing-in-itself". The thing in itself cannot be known, but we make some sort of affirmations about it. Traditional logic and science cannot accept paradoxical thinking. However, philosophy must assume it. This is the case with the Christian Trinity thesis. In a simplified way, those who reject this thesis immediately present the inconsistent logical form of the following set of sentences:

1. God = the Father
2. God = the Son
3. God = the Holy Spirit
4. The Father ≠ the Son
5. The Father ≠ the Holy Spirit
6. The Holy Spirit ≠ the Son

For Christian theology, God is one (monotheism), but it is in three persons (sentences 1, 2 and 3): Father, Son and the Holy Spirit. At the same time, the three persons are different (sentences 4, 5 and 6). But sentences 1, 2 and 3 are successively denied by sentences 4, 5 and 6. From the perspective of classical logic, the set of sentences expressing the Christian Trinity is inconsistent (Bohn 2011). For opponents of the Christian Trinity, the discussion may end here.

Yet, another sentence should be added to the set mentioned above, one about the monotheistic character of Christianity. Because the Trinity is in unity, this means there is one God instead of three. Therefore, whatever the solution is, for or against the Trinity, it will have to be compatible with the monotheistic character of the Christian faith:

7. There is one and only one God.

A representation, with the help of a triangle (Lobovikov 2015) of the Christian Trinity, can help us better understand what has been said before (see Fig. 1).

This triangle of the Christian Trinity allows us to "visualize" the reported logical inconsistency. This inconsistency is for some critics the clear sign of the illogical and irrational character of the Christian faith. But the non-contradictory character of the Christian Trinity can be demonstrated even with the means of classical logic. As we know, a contradiction between two sentences occurs when one asserts and the other denies something at the same time, about the same thing and the same aspect. For example, we would have a contradiction if we argue that
"God is a being" and that "God is three beings"; or if we argue that "God is a person" and that "God is three people". But the Trinity does not say that. It tells us that God is a being in three people, that is, God is one from a certain perspective and He is in three people, from another perspective. Since the perspective is different, the contradiction does not appear anymore.

![Diagram of the Christian Trinity]

Figure 1

The demonstration requires specific tools, other than those of classical logic. There are many older or newer attempts that try to remove the thesis of the inconsistency of the Christian Trinity. There is often a call for mathematical tools. As far as we are concerned, we will appeal to co-inherence and the nesting relationships specific to the Christian Trinity, as they appear especially in Augustine’s work.

2. "Heaven of Heavens" as the principle of free totality

Referring to the Christian Genesis and the Trinity, Augustine, in the twelfth Book of the Confessions, points out that God, when He made heaven and earth, He meant doing something great and something small, so He made a great heaven, and petty earth. That "heaven of heavens" is God’s, and the earth of men. The "heaven of heavens" was made at first as a rational creation that shares eternity. "And therefore the Spirit, the Teacher of Thy servant, when It recounts Thee to have In the Beginning created heaven and earth, speaks nothing of times, nothing of days. For verily that heaven of heavens which Thou createdst in the Beginning, is some intellectual creature, which, although no ways coeternal unto Thee,
the Trinity, yet partaketh of Thy eternity, and doth through the sweetness of that most happy contemplation of Thyself, strongly restrain its own changeableness; and without any fall since its first creation, cleaving close unto Thee, is placed beyond all the rolling vicissitude of times” (Augustine 2005, 225).

We advance the hypothesis that Augustine's metaphor "heaven of heavens" has a foundational role in the logical plane of explanation. It is not about the Aristotelian logic, but about a more subtle one, about the one of totalities. In this case, Augustine points out that in the "heaven of heavens", reason does not know "in part", but it knows everything suddenly, entirely. We are dealing with a pre-existing totality. Augustine's thought of "heaven of heavens" as pre-existing totality is extremely profound. This totality shares eternity, but on the other hand, it has a special dynamics that it can master, thus overcoming the vicissitude of time: “This then is what I conceive, O my God, when I hear Thy Scripture saying, In the beginning God made Heaven and Earth: and the Earth was invisible and without form, and darkness was upon the deep, and not mentioning what day Thou createdst them; this is what I conceive, that because of the Heaven of heavens, -that intellectual Heaven, whose Intelligences know all at once, not in part, not darkly, not through a glass, but as a whole, in manifestation, face to face; not, this thing now, and that thing anon; but (as I said) know all at once, without any succession of times …” (Augustine 2005, 228).

The reason of the "heaven of heavens" metaphor is a pre-existing logical totality, where knowledge is “at once”. Then this totality is free, as it shares eternity. We cannot fail to notice the "contemporaneity" of Augustine's logical foundation. From Frege's principle of context to Quine's or Searle's "background theory", all the approaches to this logical foundation try to establish a pre-existing supposed "given".

But what is the nature of this "given"? Its nature is rational. In addition, although it shares eternity, this "given" is not substantial-static but dynamic and functional. Augustine appropriated Plato's lesson from the Sophists. Plato asks whether the ideas (a kind of "heaven of heavens," a supposed totality) can be combined and shared. The answer is affirmative. If ideas could not be combined, they could not include one another. For example, the ideas of movement and immobility would not exist (Plato 1989, 252). Or, another example: the idea of man can be associated with other ideas, such as color, size, defects etc. We do not just say that man generally exists, but that man is tall, good, bad and so on. Thus, ideas and genres are joined together, being and otherness include each other.

If this assumed totality, which is a foundation in logical perspective, were only thought of as substantial and categorical, then we would remain in the Aristotelian logical framework. Such a framework loses the functional perspective of the pre-existing totality, a perspective that Plato
does not ignore. Since ideas share one another and can be in co-inherence and co-penetrating relationships, it follows that, logically, a subject can be associated with several attributes, as in the example above: a high man, a good man, a bad man etc. Emphasizing the substantial and categorical perspective, Aristotle’s logic postulates the contradictory opposition. With emphasis on the dynamic and functional perspective, Plato’s logic admits contrariety. If Plato admitted contradictory alterity, then contradictory ideas would no longer be able to include one another. Instead, in a contrariety relationship, participation is possible. In this perspective, for example, Non-Being is not the contradictory opposite of Being, but it is other than this Being.

In a totality, as in a founding field, we find a unity of identity and difference (Hegel), a co-belonging of the elements, because by participating, any "a", for example, is simultaneously "a" and "other than a" (Biriş 2007, 85). In addition, totality is not a generalization in the spirit of the Aristotelian abstraction, but rather a particularity, a "personalization" that does not cancel the individual, but it highlights in instantiation. In this sense, the Christian God is not an abstraction, but a divine Being in three persons, in whom He instantiates.

Let’s go back to Augustine’s example. He repeatedly mentions that, according to the Scriptures, at first God made heaven and earth (en arhé suggests that God is before any movement, any creation). Yet, Augustine immediately adds that the true "beginning" is the "heaven of heavens", that is the supposed totality, the totality of the foundation of the explanation, as we have suggested. Let us consider that heaven is "a" and earth is "b". These two entities are logically preceded by the "heaven of heavens" entity, which we can define as "c". But entity "c" encompasses (as inherent) "a" and "b": \( c = [a, b] \).

This totality with a founding role functions as a principle, which we can call the principle of free totality (Prinzip der losen Totalitäten) (Dufour 2005, 293):

8. PFT: \[ a \neq b \rightarrow \exists c (c = [a, b] \land c \neq a \land c \neq b) \]

According to this principle, the supposed totality can exist logically even in the absence of the entities, which can be updated by decisional act, as in the Christian model of creation (And God said, "Let it be light!"). For Augustine, this principle (principium sine principio) is about "one omnipotent God, and that he is the tripotent [ tripotentem ] Father and Son and Holy Spirit" (Ayres 2010, 27).

3. The co-inherence of persons in the Christian Trinity

Considering the sentences 1), 2) and 3), we can say, in agreement with Vern S. Poythress (Poytress 1995) that the understanding of the Trinity "involves three aspects inextricably. First, there is classification. Each
Person of the Trinity is classified as God. Second, there is instantiation. Each Person is particular, an “instantiation” of God, distinct from the other Persons. Third, there is an associational aspect. Each Person exists in association and communion with the other Persons (“the Word was with God”). The Aristotelian logic concentrates predominantly on the first aspect. The logic of the Trinity also includes the following two aspects: instantiation and association, which means, in our terminology, co-inherence and co-habitation.

The Christian God is an over-categorical term. He is both substance and relationship. The unique divine substance is in relation to itself and to the interpersonal relationships within it: the Father, the Son, and the Holy Spirit. The network of these relationships is the Trinity. The Father expresses the relationship of paternity, the Son expresses the relationship of filiation, and the Holy Spirit expresses the relationship of process. Their common name is the person. Thus, although it may seem paradoxical for traditional logic, the Trinity can be expressed as follows:

1 + 1 + 1 = 1        Father + Son + Holy Spirit = God (one)

Let us consider co-inherence. For the logic of the Trinity, this relationship is essential because it refers to the revelation of God Himself. The co-belonging of entities in the free and founding totality (as we have seen) also expresses co-inherence. Logically, in the history of philosophy, the idea of co-inherence is fruitfully used when, for Leibniz, it is clear that the subject of a judgment can encompass all predicates (including the ones possible). Thus, the logical content of the subject reveals the inherence of the predicates (Gallerand 2013):

9. Si B erit in A, A + B = A

Yet, inherence does not fail in tautology here? All the complications arise from the principle of identity. Aristotle, as we know, formulated a rich theory of identity, emphasizing the importance of identity in genus and numerical identity. The logic of the Trinity needs a theory of identity paired with difference because this is the only way in which the instantiation of persons can be explained. If sentences 1, 2 and 3 define the identity of the God-Father, God-Son and God-Holy Spirit, the following three sentences (4, 5 and 6) define the differences. The set of sentences seems paradoxical, as we have seen, if we look at it from the perspective of the Aristotelian logic. However, taking into account the principle of free totality (PFT) (formulated by Augustine), we can see that the founding totality also includes differences (sentence 8).

These aspects can be understood more easily by using arithmetical examples. For example, we claim that 1 = 1, on the one hand, because the entity "1", on the left of the sign "=", and entity "1", on the right of the sign "=" are the same size; on the other hand, it is equally true that 1 ≠ 1, since the entity "1" on the left is not the same entity as "1" on the right (this being another entity, distinct from the first one). Following the Aristotelian logic, one might immediately interpret that we have a
contradiction. However, it is not a contradiction because, in the first case, identity has a sense of numerical equality, while in the second case the sense of inequality does not refer to the magnitude, but to the different qualities of the entities. Similarly, considering the example given by Frege: $4 \times 2 = 11 - 3$. Here we have a numerical magnitude identity ($8 = 8$), but it is not an identity if we take into account that the left operation of the sign "=" is multiplication and the right one is subtraction.

Therefore, for the logic of the Trinity, a mathematical-constructive model of logic is more useful than a naturalist one, of genres and species (adopted by Aristotle). The free totality ("heaven of heavens") as a founder cannot be understood without instances. Human beings understand through instances. That is why God can only be understood through the instances of the Trinity, through the three persons: the Father, the Son, and the Holy Spirit. Consequently, the logic of the Trinity must be a personalist logic.

Nevertheless, in this way, logic must also assume the subjective dimension of the faithful man. This was very well understood by Augustine. He is the one who sought the explanation of the mystery of the Trinity within consciousness, not in the outside world. Eckhart states that man needs to prepare his soul in order to receive the divine light. One needs a spiritual experience of the soul in order to participate in the trinitarian life of God. Divine persons are an emanation of God. That is why we are talking about co-inherence. The "co" particle points out that the starting point is in the free totality of God, and that this totality logically precedes the rest of the process. The term "inherence" tells us that divine persons are included in this totality from the beginning. But it is only the instantiation of God in the three persons that makes it possible for man to understand the divine process.

For the logic of inherence we need to understand that the intra-trinitarian relationship is a model of relationship which is obtained by the decomposition of the free totality (not by composition, not by simple association). The "co" particle in "co-inherence" points out the unity of the nature of the trinitarians, and the instantiated persons indicate the differences. In other words, this "co-inference" expresses – in terms of logical quantification – on the one hand, the universal quantification, and on the other hand, the existential quantification. For example, if we mark the property of the godhead with $G$, any person of the Trinity with `$p$', the father with `$f$', the son with `$s$' and the holy spirit with `$hs$', then we can write:

a) The universal quantification: $(\forall p) Gp \equiv (Gf \land Gs \land Ghs)$. That is, $G$ (the property of the godhead) is found, in its entirety, in every person of the Trinity;

b) The existential quantification: $(\exists p) Gp \equiv (Gf \land Gs \land Ghs)$. There are three different persons in the property of divinity, each equal to the other.
4. The nesting relationship in the Trinity

If every person of the Trinity is an "instance" of God, Christian theology also maintains that Every Person exists in association and communion with other Persons. The Greek term for this community is perihoresis, mutual interpenetration or habitation. This is another way of expressing the Unity and Trinity of God. But how is this community, this mutual habitation? The answer is difficult to formulate and it may seem absurd. In the interpretation of St. John of Damascus, the persons (hypostases) are united without being mixed and they are separated from one another, which seems absurd. That is, the persons exist in others, but they do not mix. The Father shares all His being with the Son and the Holy Spirit, but the being of God remains a whole. None of the Trinity persons is missing here. Inequality between people is excluded, "because each of the three equally and wholly shares in or possesses the same essence" (Erickson 2017, 9). For Augustine all the three persons of the Trinity are "on the same level when it comes to their 'divinity'. The Son and the Spirit do not derive their divinity from the Father, but they have it in themselves" (Wisse 2011, 59).

This "habitation" of the persons of the Trinity can be interpreted in different ways, which is what Augustine also emphasized in his time: "But when he discovers and can say any thing of these, let him not therefore think that he has found that which is above these Unchangeable, which is unchangeably, and Knows unchangeably, and Wills unchangeably; and whether because of these three, there is in God also a Trinity, or whether all three be in Each, so that the three belong to Each; or whether both ways at once, wondrously, simply and yet manifoldly, Itself a bound unto Itself within Itself, yet unbounded; whereby It is, and is Known unto Itself and sufficeth to itself, unchangeably the Self-same, by the abundant greatness of its Unity, -who can readily conceive this? who could any ways express it? who would, any way, pronounce thereon rashly?" (Augustine 2005, 251).

Can one ignore the logical "absurdity" of this situation? The answer may be affirmative if we find another logical way to describe things (Lucaci 2005, 21). The biblical text invites us to understand things by analogy with human communication. In human communication a person addresses another person not only to inform him/her, but also to produce certain effects. We not only need an "informational" logic, but also an "expressive" logic, a logic of performative acts, which means that a more "social" logic is needed in the spirit of Augustine's ideas.

Naturally, logic views the validity of judgments. But these are done in a certain language, with certain terms and with some symbols. Therefore, one should begin even from the term 'the Christian "Trinity"'. This term wants to communicate divine unity (monotheism) on the one hand, and plurality of persons (three), on the other hand. Is Trinity a singular and
plural term at the same time? It depends on the type of interpretation used here. The logicians who use the language of object classes and sets often think that a "set" or a "class" is a singular term, even if the content is a plurality. Other logicians claim that plural terms should not be reduced to singular terms.

We are again in a situation similar to the ones in mathematics. For example, if a mathematician finds that "the square root of 4 is plus-or-minus two" is a singular term, should we take it seriously (See Oliver, Smiley 2013, 6)? Is it not fair to say that this is a plural term: "the square roots of 4 are plus and minus two"? The language of terms is also very confused so it is necessary to clarify the most nuanced aspects. Confusions at the level of terms induce confusions at the propositional level in the case of conjunctions (numerical or propositional) or disjunctions etc.

Thus, this relationship of "co-habitation" of the persons of the Trinity involves mutual habitation, without any mixing or "dissolution" of one another. They are together but still separated. It might seem difficult to understand, but the same Augustine clearly says that God is intelligible (Augustine 1992, 52), so the free totality of the Godhead is rational. There is something in something else in a communion; but precisely because that thing is different and it can only be in communion, it cannot be separated (Augustine 1992, 112). "Knowledge of God the Trinity means knowledge of the Father in the Son through the Holy Spirit" (Gioia 2008, 107).

We believe that a solution for the consideration of the nesting relationship in the Trinity can be outlined. Starting from the specifics of the functional terms (and the terms of the Trinitarian Persons are such functional terms, as we have seen, since they express the functions of paternity, filiation and of intra-trinitarian processuality), we can consider the term "the Christian Trinity" (in its plural sense) as a nested term. "Functional terms are special because they are nested terms, terms that have other terms as constituents" (Oliver, Smiley 2013, 83). This search is in the spirit of the recent evolution in the logic of terms. For a long-time logicians have been particularly concerned with the analysis of names, demonstratives, and descriptions. However, in the recent years, the analysis has been moving towards the specification of functional terms.

In this context, if we mark the relation of inclusion (assumed by the nesting relation) with "⊂", the plural identity with "=", and the variables with x, y, z, we can use the following formula for plural terms (Oliver, Smiley 2013, 110):

10) $\forall x (x \subset a) \rightarrow x = a$

This tells us that within the term ‘Trinity’, anyone of the persons in question lives together with the others (by inclusion), although they remain separate and are equal to one another and to the unitary Trinity that encompasses them. “Applied to the Trinity, the thing goes as follows. To count the number of Divine Persons, we assign 1 to the Father and then
2 to the Son, since He is not the same Divine Person as the Father, and we assign 3 to the Holy Spirit, for He is a Divine Person different from both the Father and the Son. But to count Gods, we assign the number 1 to the Father and to the Son and to the Holy Spirit as well, for each is the same God as the Father. Thus three Persons, one God. Trinity in unity” (Cartwright 1990).

In the modern age, Leibniz was particularly interested in a calculus of inclusion. For Leibniz (based on the principles $aa = a$ and $ab = ba$), each simple term could be represented by a prime number, and each compound term would be a product of prime numbers. This scheme can only contain positive sentences. To express negative propositions, Leibniz chose to represent each term by two numbers, one positive and the other negative (Kneale 1974, I 359). Things got too complicated. However, through inherence, from a sentence of form A is BCD, we can always infer that A is B, A is C and A is D. Assuming the idea that A is wholly in B, it is wholly in C or in D (that is, B, C and D are not only parts), Leibniz’s scheme could also be used in explaining the Christian Trinity.

Such attempts show us that the logician, like the scientist, can choose (sometimes arbitrarily) any type of interpretation or of working hypothesis. This means that the traditional subject-predicate scheme imposed by the Aristotelian logic is not absolute. The demarcation line between the subject and the predicate of a sentence may be a choice depending on the purpose of the logician. Such a principle is assumed in contemporary logic by Charles Sanders Peirce, in particular.

Let us consider the following sentence: "The Christian God lives in the Persons of the Trinity". Since the predicate of the sentence can be determined according to our choice, we can have at least the following statements: a) The Christian God lives ...; b) The Christian God lives in the Persons ...; c) The Christian God lives in the Persons of the Trinity. The statements a) and b) are more or less saturated. Instead, statement c) expresses the entire sentence, which means that, from Peirce's perspective, in a sentence, copula may be part of the predicate, the whole sentence forming a predicate, that is, an undecomposed unit. This way of understanding can also help in the logic of the Trinity.

For Peirce, in a broad sense, logic is equivalent to semiotics. The triadic relationship is fundamental in Peirce’s semiotics: “A Sign, or Representamen, is a First which stands in such a genuine triadic relation to a Second, called its Object, as to be capable of determining a Third, called its Interpretant, to assume the same triadic to its Object in which it stands itself to the same Object. The triadic relation is genuine, that is its three members are bound together by it in a way that does not consist in any complexus of dyadic relations” (Peirce 1960, 156).

Signs are divided into three categories: icons, indices and symbols. A parallel between the conception of Peirce and that of Augustine brings an interesting perspective here: “As foundation for his theory of scriptural
interpretation, wherein meaning is not limited to the literal, Augustine brought words under the general category of signs, after having declared that signs are that from which we learn about things. Thus he needed a definition of 'sign', *signum*, that applied equally to words and to signs normally so-called. A sign, he said, is 'a thing that causes us to think of something beyond the impression that the thing itself makes upon the senses'... This formula was new; it succeeded in encompassing both words and natural signs. And, like Peirce's conception of a sign, it is triadic. A sign is anything that causes one to think (such a thought is an interpretant) of something else (the sign's object)” (Short 2007, 23-24).

In addition, through the three categories of the world (*Firstness*, *Secondness* and *Thirdness*), Peirce seems to offer us a theological cosmology. *Firstness* can designate a realm of universals, a "heaven of heavens" (as for Augustine), but it can also be felt as some simple unitary consciousness. *Secondness* is the actuality of here and now. And *Thirdness* is the "nexus of Real Occasions which illustrate the Eternal Objects" (Britton 2005, 450). Peirce's triad is irreducible and holistic (Biris 2010, 32-34).

Using Peirce's ideas, we can say that the nesting relationship in the Christian Trinity designates three persons as equivalent to the three concepts of relationships: paternity, filiation, and processuality. As a process of permanent interaction, the co-habitation relationship is a transitive one. In Zemach's words, the relation "is-in-the-same-world" is transitive (Zemach 2013, 4). The persons of the Trinity are in the same world as divine persons.

Evidently some may dislike Peirce's logic, especially those who claim maximum rigor, and this is because Peirce uses the "logic of vagueness". As Peirce suggests, because our thinking can only work through signs (according to the triadic structure mentioned above), vagueness is coexistent with human thinking. Wittgenstein pleads for the same idea in The Blue Book where he claims that a sentence takes its meaning from the sign language system. The Christian Trinity cannot escape this vagueness. For Peirce, any concept or predicate is vague: a concept may be vague by its extension (by its object); also in intensity (because it is influenced by the interpretant). In fact, vagueness arises from the nature of the sign, which is implicitly enrolled in a semiotic process (Chauviré 1995, 16). This vagueness especially touches the existential quantification because variables can often be indefinite. Yet, the terms of traditional logic are not really names, but unsaturated concepts, therefore they are vague. At the same time, religious terms are especially metaphorical (see also Kaufmann 1969, 173), and this requires an ample process of conceptual refinement for logical treatment (Biris 2009). This is what the present study eventually tries to formulate with regard to the logic of the Trinity.
References


